Test 1 Review Answers

MTH 1400 Precalculus

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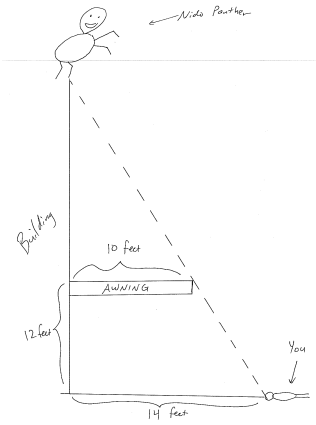
1. Jane invests \$1000 in a bank account at a certain interest rate, and \$3000 at an interest rate that is 3% higher. At the end of one year, she has earned \$270 from the two accounts. At what rate did she invest the \$1000?

   Ans: 4.5%

2. HPU is constructing a new math building, and of course they want to make it as beautiful as possible. At the top of the building in the front entranceway they decide to build a giant panther statue with the head of Nido Qubein. You want to find out how tall the building is, so you do the following: From the front entrance, you walk out and lay on the ground at the point where your line of sight connects the awning with the base of the statue. If your head is 14 feet from the bottom of the building, the awning is 12 feet high, and the awning sticks out 10 feet horizontally from the building, find out how tall the building is.

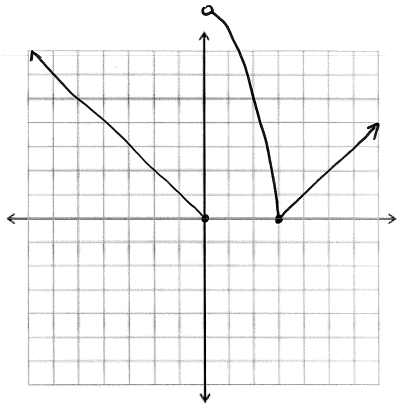


   Ans: 42 feet.

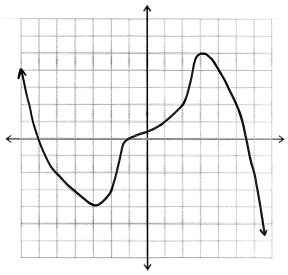
3. A hotel charges \$60 a night for the first three nights, and \$45 each additional night thereafter. Create a piecewise function $C(x)$ to represent the cost of staying x nights.

   Ans: $C(x) = \begin{cases} 60x & \text{if } 0 \leq x \leq 3 \\ 180 + 45(x-3) & \text{if } x > 3 \end{cases}$

4. Sketch a graph of the piecewise function $g(x) = \begin{cases} -x & \text{if } x \leq 0 \\ 9 - x^2 & \text{if } 0 < x \leq 3 \\ x - 3 & \text{if } x > 3 \end{cases}$



5. Find all local maximums and minimums (and where they occur), as well as the intervals where the function is decreasing and where it is increasing.



Ans: Local minimum of -4 at $x = -3$, local maximum of 5 at $x = 3$. The function is decreasing on the intervals $(-\infty, -3)$ and $(3, \infty)$ and increasing on the interval (-3,3).

6. Find all local maximums and minimums of the function $g(x) = x^3 + 7x^2 - 30x$. Round to the nearest 0.1.

   Ans: Local minimum of -26 at $x = 1.6$, local maximum of 216.8 at $x = -6.3$.

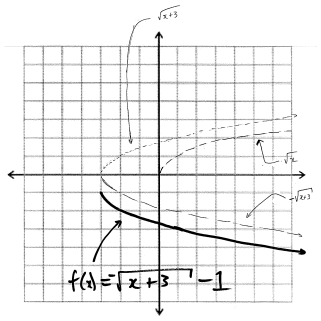
7. Find the average rate of change from $x = -5$ to $x = 4$ for the function in question 5.

   Ans: $\frac{4-(-2)}{4-(-5)} = 6/9 = 2/3$

8. Find the average rate of change from $x = 1$ to $x = 1 + h$ for the function in question 6.

   Ans: $-13 + 10h + h^2$

9. Starting with the graph of $y = \sqrt{x}$, shift, flip and/or stretch the graph to find the graph of $y = -\sqrt{x+3} - 1$.



10. Let $f(x) = \dfrac{1}{\sqrt{x}}$, $g(x) = x^3$, and $h(x) = x^2 + 2x + 3$. Find the composition of functions $g(h(f(x)))$. Simplify your answer if possible.

    Ans: $\left(\dfrac{1}{x} + \dfrac{2}{\sqrt{x}} + 3\right)^3$ or $\frac{1}{x^3} + \frac{6}{x^{2.5}} + \frac{9}{x^2} + \frac{26}{x^{1.5}} + \frac{63}{x} + \frac{54}{\sqrt{x}} + 27$

11. Graph each function to confirm that it is one-to-one. If it is not, restrict the domain so that it is one-to-one. Then find the inverse for each function.

    (a) $f(x) = \dfrac{2-3x}{4+x}$

    Ans: One-to-one. The inverse is $f^{-1}(x) = \dfrac{2-4x}{x+3}$

    (b) $f(x) = \dfrac{1}{2}(x-7)^2 + 3$

    Ans: Not one-to-one. Two answers- if the domain is restricted to $(-\infty, 7]$, then the inverse function is $f^{-1}(x) = -\sqrt{2(x-3)} + 7$. If the domain is restricted to $[7, \infty)$, then the inverse function is $f^{-1}(x) = \sqrt{2(x-3)} + 7$.

12. The ... charges \$600/person if exactly 20 people sign up for the cruise (which gives a total revenue of $600 \cdot 20 = 12,000$ dollars) . However, if more than 20 people sign up (up to the maximum capacity of 90) for the cruise, then each fare is reduced by \$4 for each additional passenger. ... Assuming at least 20 people sign up for the cruise, let x be the number of passengers over 20 who sign up for the cruise. Answer the following questions:

    (a) Construct a quadratic function $r(x)$ to represent the total revenue in terms of x. **Ans**:$r(x) = (600-4x)(x+20)$. The 600-4x is the price per passenger, and the $x+20$ is the total number of passengers.

    (b) Find the number of passengers that would give the *maximum* amount of revenue for the owner of the yacht. **Ans**: Multiplying out $r(x)$ gives $r(x) = -4x^2 + 520x + 12000$. Then $x = \frac{-b}{2a} = -520/-8 = 65$, so the number of passengers who would give a maximum revenue is 65+20=85.

    (c) What is the maximum revenue possible? **Ans**: $r(65) = 340 \cdot 85 = 28,900$ dollars.

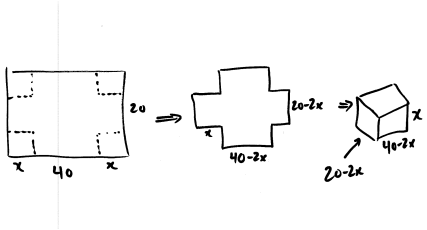
    (d) Explain how you could find (b) and (c) using a graph of $r(x)$, and how you would find them without using a graph. **Ans**: Without using a graph it is how I did it above, finding -b/2a and then substituting back into the equation. From the graph, you would find the local maximum point on the graph. The x-value is the answer for (b), the y-value is the answer for (c).

13. Find the quotient and remainder for the division $\dfrac{x^5 - 2x^4 + x^3 - 3x + 1}{x^2 + 4x - 1}$

Ans: Quotient is $x^3 - 6x^2 + 26x - 110$, remainder is $463x - 109$.

14. An open box with a volume of 1500 cm^3 is to be constructed by taking a piece of cardboard 20 cm by 40 cm, cutting squares of length x from each corner, and folding up the sides. Show that this can be done in two different ways and find the exact value of x in each case. Note that "exact" means that I do not just want a decimal approximation!

(a) Draw a diagram of the situation. Start with a 20×40 rectangle, then show how the corners are cut out, then show how it is folded up into a box. **Ans**:



(b) Write an equation (in terms of x) that represents the volume of box, and then move stuff to one side so that you have a polynomial equal to zero. **Ans**: Volume$= l \cdot w \cdot h = (40 - 2x)(20 - 2x)(x)$. Since the volume is 1500, we have the equation

$$1500 = (40 - 2x)(20 - 2x)(x) \text{ or } 0 = 4x^3 - 120x^2 + 800x - 1500$$

(c) Use graphing and/or factoring techniques to find all the zeroes of the polynomial from part (b). **Ans**: Graphing $f(x) = 4x^3 - 120x^2 + 800x - 1500$, it looks like $x = 5$ is a zero. Polynomial long division gives $f(x) = (x-5)(4x^2 - 100x + 300)$. Using the quadratic formula on $4x^2 - 100x + 300$ gives zeroes of $(100 \pm \sqrt{5200})/8$. One of those roots is too large, though, because $x = (100 + \sqrt{5200})/8 \approx 21.5$. Our width of the rectangle is only 20, so x cannot be bigger than 10 and so we toss out that zero as a possible solution. Thus the two possible answers are $x = 5$ and $x = (100 - \sqrt{5200})/8$

15. Find all zeroes for the following polynomials, both real and complex:

(a) $x^5 + 5x^3 - 36x$

Ans: $x^5 + 5x^3 - 36x = x(x^4 + 5x^2 - 36) = x(x^2 + 9)(x^2 - 4) = x(x + 3i)(x - 3i)(x + 2)(x - 2)$ Zeroes are 0, $\pm 3i$, ± 2.

(b) $x^5 - 2x^4 + x^3 - 8x^2 + 16x - 8$ (Hint: try factoring by grouping- You will have to group in a different manner than previous problems, though, possibly rearranging some of the terms).

Ans: $x^5 - 2x^4 + x^3 - 8x^2 + 16x - 8 = (x^5 - 2x^4 + x^3) - 8x^2 + 16x - 8 = x^3(x^2 - 2x + 1) - 8(x^2 - 2x + 1) = (x^3 - 8)(x^2 - 2x + 1) = (x^3 - 8)(x - 1)^2 = (x - 2)(x^2 + 2x + 4)(x - 1)^2$. Using quadratic formula on $x^2 + 2x + 4$ gives zeroes=2, 1, and $-1 \pm i\sqrt{3}$.

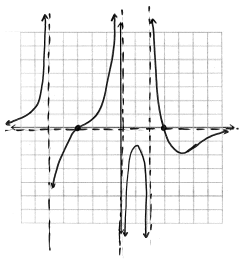
16. (a) Find all x and y intercepts and all vertical and horizontal asymptotes for the rational function

$$f(x) = \frac{-2(x-3)(x+3)}{x(x+5)(x-2)}$$

Ans: x-intercepts at $x = 3$ and $x = -3$, y-intercept does not exist. Vertical asymptotes at $x = 0$, $x = -5$ and $x = 2$; Horizontal asymptote at $y = 0$.

(b)Sketch the graph of $f(x)$.

Ans: You need to look at how the function acts as x approaches the asymptotes to know where the graph goes. For example, as x approaches 2 from the right the numerator will be $- \cdot - \cdot +$ (which is positive) and the bottom will be $+ \cdot + \cdot +$ which is positive, so the graph is going to positive infinity as x approaches 2 from the left.



17. Bob invests \$10,000 in a bank account at 4% interest, compounded continuously. How much money will he have in the account after 5 years? Round to the nearest dollar. **Ans**: \$12,214

18. Radioactive iodine is used as a tracer to diagnose certain thyroid gland disorders. It decays in such a way that the mass (in grams) remaining after t days is given by the exponential decay function, with $P = 6$ and $r = -0.087$. Answer the first two questions *without* using a calculator. You will need a calculator to answer the third question.

(a) How much of the iodine is present initially? **Ans**: 6 grams
(b) How much iodine will be left in the body over the long run (that is, as t goes to infinity)? **Ans**: 0 grams
(c) How long does it take for the half of the iodine to leave the body? **Ans**: 7.96 days

19. Use the definition of logarithm to solve the equations. You should be able to do these without a calculator.

(a) $\log_4 2 = x$ **Ans**: $1/2$
(b) $\log_4 x = 2$ **Ans**: 16
(c) Evaluate $\log_5 100 - \log_5 10 + \log_5 5 - \log_5 2$ **Ans**: 2

20. Use laws of logarithms to completely expand the expression $\ln\left(\dfrac{e^x}{x(x^2+1)(x^4+1)}\right)$.

Ans: $x - \ln x - \ln(x^2+1) - \ln(x^4+1)$